# Design of Set Point Regulators for Processes Involving Time Delay

An efficient frequency domain methodology for the design of set point regulators for high-order process models involving relatively large pure time delay is presented. The unique advantage of the methodology is that it enables the designer to rapidly visualize the structure of the required controller as soon as the time and frequency domain specifications are stipulated for the closed-loop system. Optimizing the parameters of an appropriate structure improves the robustness of the controller. The method does not require order reduction of the original model, and time delay is handled directly. Also, only output feedback is utilized.

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## Introduction

The presence of large time delays and uncertainty in model parameters poses a serious challenge to the design of robust feedback controls for chemical processes. Adaptive and optimal control theories have been applied to the solution of process control problems by numerous investigators. In spite of the availability of a multitude of algorithms and a variety of control computer hardware, the process industry has been reluctant to use them. Even today, the norm is the conventional type (PI, PID, etc.) controllers and all others are by and large exceptions. The underlying reason for this is controller complexity.

The sophisticated controllers (designed via optimal control theory or pole placement) generally require information from all the states of the process. The complexity can be visualized from the control law, $u(t)$, for zero set point regulation of a time invariant process (Manitius, 1984):

$$u(t) = \Gamma X(t) + \int_{-\tau}^{0} \phi(\theta) X(t + \theta)\, d\theta$$

where $X$ is the process state vector, $\Gamma$ the feedback coefficient vector, $\tau$ the time delay, $\theta$ a dummy variable of integration, and $\phi$ a vector function of time defined in terms of the process parameters. The major attribute of the above control law is that it can handle large values of $\tau$. One therefore accepts its complexity only in situations where other, simpler designs are not satisfactory.

Use of simplified process models unfortunately give rise to states that may not have a direct and meaningful interpretation for measurements. Consequently, for applying state space techniques an accurate and sophisticated model must be developed first. Although, in theory this can be accomplished by a rigorous interpretation of the chemical and physical phenomenon, in practice it becomes unwieldly for large-scale systems (Edgar, 1980). Furthermore, the task of determining the precise location of the sensors is not trivial.

When the states are not accessible, in theory they can be estimated from the output by means of the process model (Luenberger, 1971). However, these estimates are often unreliable since the process model parameters and sometimes its structure have large uncertainties. Even when the process model is known fairly accurately, the increased controller complexity has been a major deterrent to its widespread acceptability.

A similar situation arises when LQG (linear-quadratic Gaussian) procedures (Athans, 1971; Stein and Athans, 1987) are employed. Since the method is basically for rational transfer functions (TF), an irrational process TF must be converted approximately to a rational form. This will usually result in a TF of uncomfortably high order even when the rational part of the process model is of low order (Bada, 1984). The resulting output feedback controller will also be of high order. Therefore, to obtain practically implementable controllers, order reduction of the full order controller will be necessary (Enns, 1984). Unfortunately, the design process becomes computationally complex and the designer has to go through many iterations with several different performance indexes before satisfactory time responses are obtained. Also, the complexity of the final controller cannot be foreseen by the designer.

Recognizing the need for low-order, readily implementable controllers, Ziegler and Nichols (1942) proposed a simple PID controller for process regulation. They also devised a simple tuning procedure. A number of other simple methods have been proposed (Cohen and Coon, 1953; Murril, 1967), but they have shown little or no improvement. The Ziegler-Nichols controller settings are still the unofficial standards in the process control field. They are easy to find and give reasonable performance on most control loops. However, when time delay is significant these settings will usually result in unsatisfactory closed-loop response. Several time-consuming trials will be necessary to find suitable controller settings.

A design method using only output feedback has been reported recently (Bada, 1984). It requires a rational approximation of the irrational process transfer function. Design is accomplished by using the method of Zakian (1983). Unfortunately, the designer has to select a suitable controller structure by trial and error. Also, the design criteria are mathematically abstract. Consequently, important design requirements such as overshoot, settling time, and others cannot be incorporated directly in the design process.

The internal model control (IMC) scheme has gained popularity ever since its formal exposition in 1982 (Brosilow and Tong, 1978; Garcia and Morari, 1982; Morari and Doyle, 1986). It involves the use of a nominal process model that is explicitly in the control scheme in addition to a controller. The advantage of the IMC is that a suitable controller can be visualized quickly. However, when the process model is of high order with oscillatory response characteristics, the complexity of the controller and the on-line implementation of the process model can be problematic. This is especially true if an analog implementation is contemplated.

The purpose of this paper is to present a simple and computationally efficient method that will directly provide robust and relatively low-order controllers for set point regulation. The method requires only output feedback. Also, only an approximate transfer function of the process is required. Unlike IMC, the process model is not used explicitly in the controller. The main advantage of the method is that design specifications in the time and frequency domains can be incorporated directly in the design procedure.

Unlike the Ziegler-Nichols method, the controller structure is not frozen to be PID. Instead, an appropriate structure is identified and subsequently its parameters are optimized. Design is carried out in the frequency domain. The approach, however, differs from the methods employing Bode plot, Nichols chart, and the like. The primary goal of these traditional methods is to satisfy certain design requirements such as gain and phase margins. Suitability of the time response of the controlled system has to be checked subsequently via simulation. This approach usually results in laborious iterations between the frequency and time domains. In contrast, the methodology in this paper will directly yield a robust low-order controller that will produce a prespecified step response for the closed-loop system.

## Controller Design Procedure

A process is to be regulated using an appropriate controller in a closed-loop fashion. The arrangement is shown in Figure 1. In general, the process transfer function $G_p(s)$ will contain a rational part, $G(s)$, and a time delay, $\tau$. The design goal is to find a low-order, rational controller $K(s)$ such that the response, $C_1(t)$, closely matches the response, $C_2(t)$, of a prespecified reference model, $G_R(s)$, expressed as $A(s)e^{-s\tau}$ in Figure 1.

It is important to note that the reference model must contain the same amount of time delay as the process model. Otherwise,

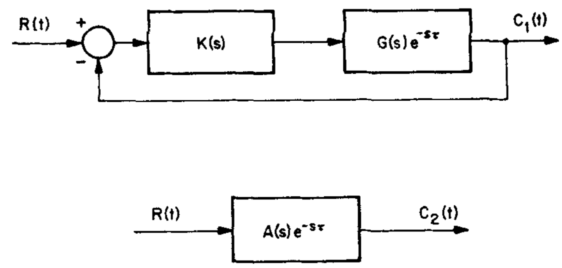


**Figure 1. System configuration for design.**

the controller either will be anticipatory or will contain unnecessary pure time delay.

The rational part, $A(s)$, of the reference model is chosen by the designer to meet certain design requirements. These include time domain requirements such as overshoot and settling time, as well as frequency domain requirements such as bandwidth, cut-off rate, and others. The reference model can also be viewed as a closed-loop system. Therefore, certain gain and phase margins for the design are indicated by the reference model.

For example, consider a second-order, critically damped reference model:

$$G_R(s) = \frac{e^{-s}}{(1+s)^2}$$

The step response of this reference model will not have any overshoot, and its settling time (to within 5% of the final value) will be approximately 6 s, including the pure time delay of 1 s. It has a bandwidth of 0.64 rad/s and a cut-off rate of40 dB/decade. If $G_R(s)$ is viewed as a closed-loop system with unity feedback, it will have a phase margin of 65° and a very large gain margin.

The controller, $K(s)$, is designed such that the match between the frequency responses of the closed-loop system and the reference model is extremely close. This will guarantee that the closed-loop system will also exhibit the same frequency response (and therefore the same time response) characteristics as the reference model. Also, the match will impart the same gain and phase margins to the design, which will give a measure of the robustness of the controller. Since these margins are made large by choosing $G_R(s)$ appropriately, the controlled response of the actual process will not deviate significantly from that of the reference model even though the process model parameters may have a large margin of error.

In order that the responses $C_1(t)$ and $C_2(t)$ in Figure 1 match, it is necessary that

$$K(s) = \frac{G_P(s)^{-1}}{G_R(s)^{-1} - 1} \tag{1}$$

Equation 1 is generally known as the synthesis equation. Its use in controller design (for $\tau = 0$) dates back several decades (Freeman, 1957; Cohen and Coon, 1953). An elegant exposition of the technique is available in papers by Peczkowski and Sain (1977, 1985), Sain et al. (1981), and Sain and Peczkowski (1985). Unfortunately, if $\tau$ is sizable and/or $G(s)$ is of high order, the resulting expression for $K(s)$ in Eq. 1 will be unsuitable for practical implementation. To solve this problem, a two-step procedure has been adopted in the past (Peczkowski and Sain, 1977). In step one, a rational reduced order model for $G_p$ is obtained. Using the reduced process model and employing algebraic manipulations on the righthand side of Eq. 1, an appropriate expression for $K(s)$ is obtained in step two.

When the plant transfer function is irrational, its conversion to a rational form will usually require a transfer function of high order. For example, it is shown by Bada (1984) that a transfer function,

$$G_P = \frac{e^{-2s}}{(1+s)(1+0.5s)}$$

when converted (using the moments matching method) to a rational form having reasonable accuracy will result in the following:

$$G_P \approx \frac{+0.019s^3 + 0.192s^2 - 0.714s + 1.0}{0.004s^7 + 0.04s^6 + 0.23s^5 + 0.84s^4 + 2.08s^3 + 3.19s^2 + 2.79s + 1.0}$$

If a low-order rational approximation is used, discrepancies in the model can cause the controller to fall short of design specifications.

The advantage of the methodology of this paper is that it eliminates the need for a reduced-order rational approximation of $G_p(s)$. The dominant controller structure and optimum parameter values are determined from the frequency response information of the "ideal" controller, $K(s)$, in the low-frequency band. This information can be generated readily from Eq. 1 using the original $G_p(s)$, and the specified $G_R(s)$. $G_R(s)$ is assumed to have a steady state gain of unity.

The most important step in the design is the identification of an appropriate structure(s) for the controller. This task begins with the determination of the number of poles, $\nu$, at the origin for $K(s)$. This is accomplished readily by inspecting $G_R(s)$ and $G_p(s)$. Let

$$B(s) = \frac{A(s)}{1 - A(s)} \tag{2}$$

If, at the origin, $B(s)$ has $\nu_1$ poles and $G_p(s)$ is known to obtain $\nu_2$ poles, then:

$$\nu = \nu_1 - \nu_2 \tag{3}$$

For example, let:

$$G_P(s) = \frac{e^{-2s}}{(1+s)(1+0.5s)}$$

and let the reference model be:

$$G_R(s) = A(s)e^{-2s} = \frac{1}{(1+2s)^2}e^{-2s}$$

then:

$$B(s) = \frac{1}{s(4+4s)}$$

Clearly, $\nu_1 = 1$, $\nu_2 = 0$, and consequently, $\nu = 1$.

Having determined $\nu$, the Nyquist plot of

$$L(j\omega) = (j\omega)^\nu \times K(j\omega) \tag{4}$$

must be examined instead of that of $K(j\omega)$ of Eq. 1 to identify the remaining structure. This is to isolate the poles of $K(s)$ at the origin so that they do not obscure the behavior of $K(j\omega)$ at low frequencies.

The dominant structure of $L(s)$ may be identified readily by logically translating the increasing/decreasing trends in the magnitude and phase angle of its Nyquist plot at low frequencies. This is illustrated in the examples.

Having determined the dominant structure of $L(s)$, it is expressed as a polynomial ratio, $P(s)/Q(s)$. The coefficients of the polynomials, $P$ and $Q$, are determined by the method of error minimization in the frequency domain. The specific method employed is explained in the next section.

The initial structure is expanded if the quality of the closed-loop transfer function is not satisfactory. Expansion is done by adding one or more pole-zero pairs to the dominant structure until an acceptably low value for the index of quality (discussed in the following section), is reached. An independent check on the closed-loop stability, using the Nyquist criterion, is made before accepting any controller.

## Parameter Optimization

Let $L(j\omega)$ be approximated as:

$$L(j\omega) \approx \frac{a_0 + a_1(j\omega) + \cdots + a_m(j\omega)^m}{1 + b_1(j\omega) + \cdots + b_n(j\omega)^n} = \frac{P(j\omega)}{Q(j\omega)} \tag{5}$$

The problem then is that of determining optimum values for the coefficients, $a_0, \ldots, a_m, b_1, \ldots, b_n$. Note that the values of $m$ and $n$ are specified by the structure chosen for $K(s)$. The approximation error at $\omega_k$ is:

$$\epsilon_k = L(j\omega_k) - \frac{P(j\omega_k)}{Q(j\omega_k)} \tag{6}$$

Let $N$ be the number of frequency points (usually 100 or more) at which $L(j\omega)$ has been evaluated using Eqs. 1 and 4. Then the error index to be minimized is:

$$J = \sum_{k=1}^{N} W_k |\epsilon_k|^2 \tag{7}$$

where $W_k$ is an optional frequency-dependent weighting function.

The criterion employed in choosing $W_k$ is that it should decrease with frequency, the decrease becoming sharp beyond a certain frequency, say $\omega_c$. The obvious implication of this criterion is that a weighing function can facilitate relatively closer matching in the important regions of low and mid frequencies. A simple form for $W_k$ that is found to be effective in a variety of cases is:

$$W_k = \frac{1}{(1 + \omega_k/\omega_c)^p} \tag{8}$$

where $p$ is an integer $\geq 0$. The cutoff frequency $\omega_c$ may be chosen as the bandwidth of $A(j\omega)$. It is found via extensive numerical experimentation that if the value of $a_0$ is constrained to be equal to the steady state gain of $L(j\omega)$, a reasonable choice for $W_k$ can simply be unity. The steady state gain of $L(j\omega)$ can be determined easily by evaluating the limit as $\omega \rightarrow 0$.

### *Quality criterion*

Although the accuracy provided by a controller is judged ultimately by simulating the closed-loop system, a simple criterion in the frequency domain is essential to quickly and systematically rank the various possible reduced-order controllers and to determine the point of diminishing return while increasing their order.

For an index of quality to be meaningful, it must incorporate a measure of closeness of the frequency response of the closed-loop system, $H(j\omega)$, with that of the reference model, $G_R(j\omega)$. With this as the guideline, the following normalized index of quality has been adopted:

$$\eta = \frac{\int_0^\infty |\Delta|^2 \, d\omega}{\int_0^\infty |A(j\omega)|^2 \, d\omega} \times 100 \tag{9}$$

where $|\Delta| = |G_R(j\omega) - H(j\omega)|$

The upper limit of $\infty$ for the integral in Eq. 9 is interpreted as a high value of $\omega$ in the numerical evaluation of $\eta$. Note that in view of the Parseval theorem, $\eta$ indicates the relative energy of the error between the impulse responses of $H(s)$ and $G_R(s)$.

As mentioned in the previous section, the order of the controller is increased until a low value (typically, lower than 0.1%) for $\eta$ is attained. This is to guarantee that the frequency response of $H(s)$ is sufficiently close to that of $G_R(s)$.

### *Algorithm for parameter optimization*

Several techniques are available (Stahl, 1980; Sanathanan and Tsukui, 1974) to obtain the optimum values for the parameters $a_i$ and $b_i$ once the values of $m$ and $n$ in Eq. 5 are specified. Among them, the matrix adaptation technique in Stahl (1980) is found to be the most effective. The algorithm has been designed such that certain parameter values (for example, $a_0$, the steady state gain) can be frozen. The technique has the distinct advantage over direct search methods (Nelder and Mead, 1965; Luus, 1980) that the user does not have to supply initial guess values for unknown parameters or to define a search space. The algorithm also has excellent convergence properties.

## Design Examples

### *Example 1*

Consider the design of a set point regulator for the following process model:

$$G_P(s) = \frac{K_p e^{-\tau s}}{(1 + as)(1 + bs)}$$

The steady state gain, $K_p$, can usually be measured fairly accurately. Large uncertainties, however, will be present in the dynamic parameters $\tau$, $a$, and $b$. For convenience, let $K_p = 1.0$, and assume nominal values for $\tau$, $a$, and $b$ of 2.0, 1.0, and 0.5, respectively. These quantities can be in any time units (seconds, minutes, etc.); they are assumed to be in seconds for this example.

Design is performed for the nominal model. Robustness is demonstrated by considering large variations in the dynamic parameters. In order to illustrate the influence of the reference model on the controller structure, two reference models with different response times are considered.

$$\text{Case 1: } G_{R_1}(s) = \frac{e^{-2s}}{(1 + 2s)^2} \quad \text{(settling time} \approx 15 \text{ s)}$$

$$\text{Case 2: } G_{R_2}(s) = \frac{e^{-2s}}{(1 + s)^2} \quad \text{(settling time} \approx 7 \text{ s)}$$

Since $B_1(s)$ and $B_2(s)$, Eq. 2, each have a pole at the origin, and since $G_p(s)$ does not have a pole at the origin, the required controllers will each have a pole at the origin, making $\nu = 1$ for each case. Therefore, define:

$$L_1(s) = s \times K_1(s)$$
$$L_2(s) = s \times K_2(s)$$

where $K_1(s)$ and $K_2(s)$ are the required "ideal" controller transfer functions, Eq. 1. The dominant structures of $L_1(s)$ and $L_2(s)$ are obtained by examining their Nyquist plots as given in Figure 2. Consider $L_1(j\omega)$ first. The pattern of the Nyquist plot as $\omega$ increases is typical of a phase-lead structure. Whereas, in the case of $L_2(j\omega)$, the real part decreases and the imaginary part increases as $\omega$ is increased. This indicates that $L_2(s)$ must have a

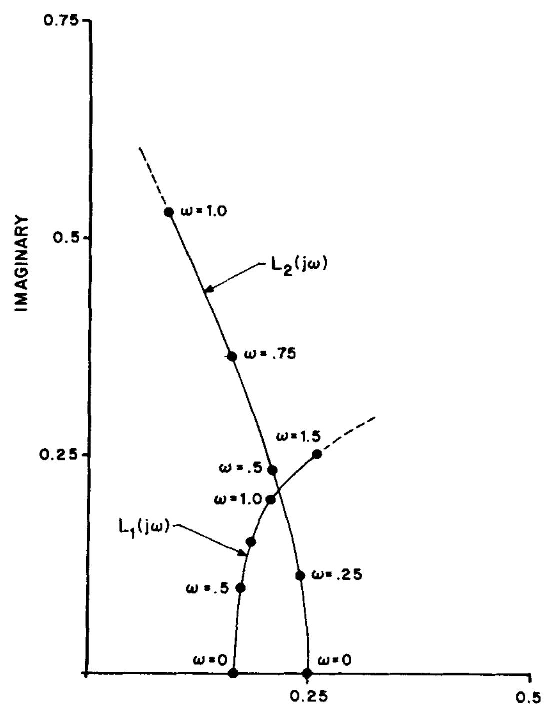


**Figure 2. Nyquist plots of $L_1(s)$ and $L_2(s)$, example 1.**

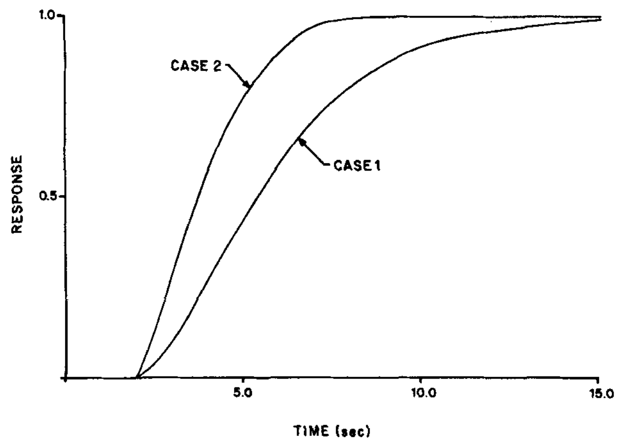


**Figure 3. Closed-loop step responses for the cases in example 1.**

double-zero structure. Therefore, $L_1$ and $L_2$ are approximated as:

$$L_1(s) \approx \frac{k(1 + T_1 s)}{1 + T_2 s} \quad \text{with } T_1 > T_2$$

and

$$L_2(s) \approx a_0 + a_1 s + a_2 s^2$$

Optimization of the parameters result in the following low-order controllers:

$$K_1(s) \approx \frac{0.1667}{s} \times \frac{1 + 1.298s}{1 + 0.115s} \quad \text{(integral and phase lead)}$$

$$K_2(s) \approx 0.4478 + \frac{0.25}{s} + 0.153s$$

Closed-loop unit step responses with each of the above controllers are given in Figure 3. They match the corresponding reference responses almost exactly. Consequently, no further additions to the controller structures are necessary.

It is important to note that the above design procedure (for either case) did not involve any trial and error whatsoever.

This example illustrates how the method enables the designer to quickly visualize the type of controller needed to duplicate any reference response. This facility, a unique feature of the present method, is extremely valuable.

The Ziegler-Nichols PID controller, $K_{Z\text{-}N}$, has also been designed for comparison (Luyben, 1973):

$$K_{Z\text{-}N} = K_c\left(1 + \frac{1}{\tau_I}s + \tau_D s\right)$$

with $K_c = K_{max}/1.7$, $\tau_I = T_0/2$, and $\tau_D = T_0/8$, where $K_{max}$ is the proportional-only gain to make the closed-loop system exhibit stable and sustained oscillations, and $T_0$ is the period of these oscillations. $K_{max}$ and $T_0$ for the nominal values of the process parameters are 1.54 and 6.54, respectively. Therefore,

$$K_{Z\text{-}N} = 0.906 + \frac{0.326}{s} + 0.740s$$

The unit step response using $K_{Z\text{-}N}$ is shown in Figure 4. It is clear that a major retuning of the controller is necessary.

Robustness of the integral phase-lead controller, $K_1(s)$, is demonstrated by examining the step response of the corresponding closed-loop system for large variations in the dynamic parameters. Figure 5 shows the step responses for $\tau = 1.0$, 2.0 (nominal), and 3.0, respectively. Figure 6 shows the step responses for $b = 0.1$, 0.5 (nominal), and 1.5, respectively. (Note that the variation in $b$ overlaps the nominal value of $a = 1.0$). Clearly, the responses do deviate somewhat from the nominal. However, there appear to be no stability problems whatsoever. A similar degree of robustness was also observed for case 2.

In general, changes in the dynamic parameters will have strong influence on the phase margin of the controlled system. However, since the reference models, $G_{R_1}$ and $G_{R_2}$, are chosen to reflect a large phase margin for the nominal system, the resulting controllers are able to accommodate large changes in the model parameters.

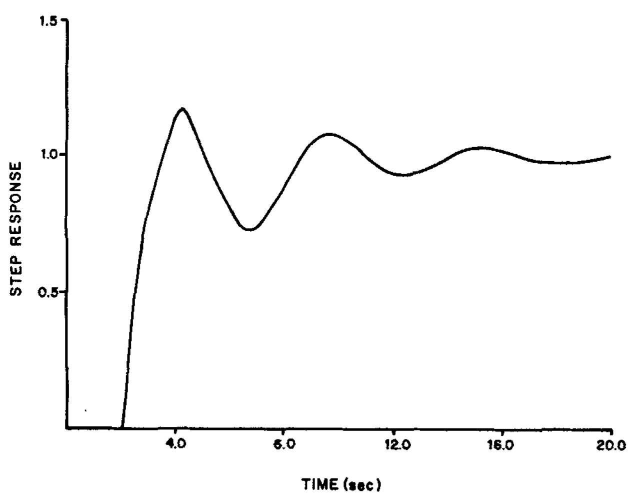


**Figure 4. Closed-loop step response, example 1, using Ziegler-Nichols PID controller.**

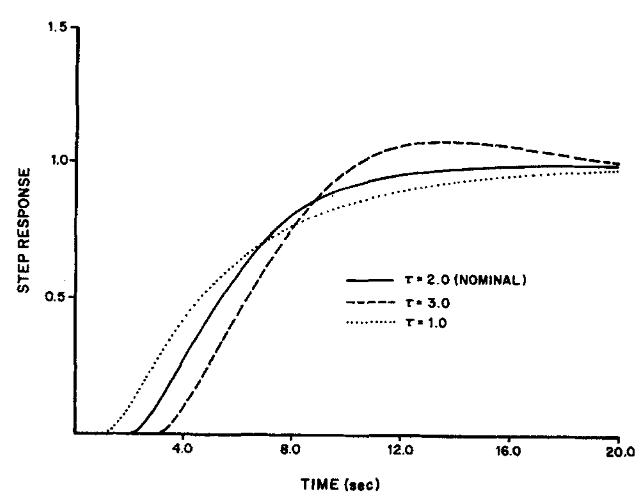


**Figure 5. Sensitivity of closed-loop response, example 1, to variations in time delay.**

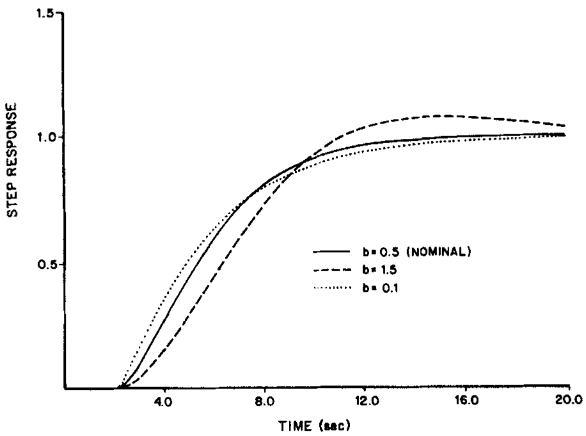


**Figure 6. Sensitivity of closed-loop response, example 1, to variations in time constant *b*.**

### *Example 2*

This example is to illustrate that the method can directly produce low-order, easily implementable controllers even though the process model is of high order with time delay. Note that the state space methods have to reduce the order of the process model to achieve reduction in controller complexity. This, as pointed out previously, is not usually accurate when the time delay is sizable.

The following eighth-order process model is considered.

$$G_p(s) = \frac{[(s + 0.6883)(s^2 + 1.421s + 1.0832) \times (s^2 - 1.6962s + 66.227) \times (s^2 + 99.4606s + 8294.213)]}{[13(s + 1.0617)(s + 1.2812)(s + 1.8152) \times (s + 3.3039)(s + 8.6994)(s + 20.1418) \times (s^2 + 1.314s + 22.043)]} \times e^{-2s}$$

The oscillatory step response of the open-loop system is given in Figure 7.

Order reduction of just the rational part of $G_p$ using the balanced truncation method (Glover, 1984) shows that at least a fourth-order reduced model will be necessary. This, however, is of no consequence in the method presented, since the controller structure is not (unlike the state-space methods) determined by the model alone.

A simple reference model is specified even though this plant model is relatively more complex than the previous example.

$$G_R(s) = \frac{1}{(1 + 2s)^2} e^{-2s}$$

The expression for the "ideal" controller is again given by Eq. 1 and it will have a pole at the origin. Therefore, the Nyquist plot of $L(s) = sK(s)$ as given in Figure 8 is considered. Note that its behavior at low frequencies is considerably different from that in the previous example. It resembles the signature of a phase-lag compensator. Therefore, initially, $L(s)$ is approximated as:

$$L(s) \approx \frac{k(1 + T_1 s)}{1 + T_2 s} \quad T_1 < T_2$$

Optimizing the parameters, the following ½ approximation for $K(s)$ is obtained.

$$K\left(\frac{1}{2}\right) = \frac{0.1667}{s} \times \frac{1 + 0.1441s}{1 + 0.8121s}$$

The unit step response of the closed-loop system with the above controller, as well as that of the reference model, is given in Figure 9. If the minor discrepancy is not tolerable, the controller structure may be expanded to ⅔ by adding a pole-zero pair to

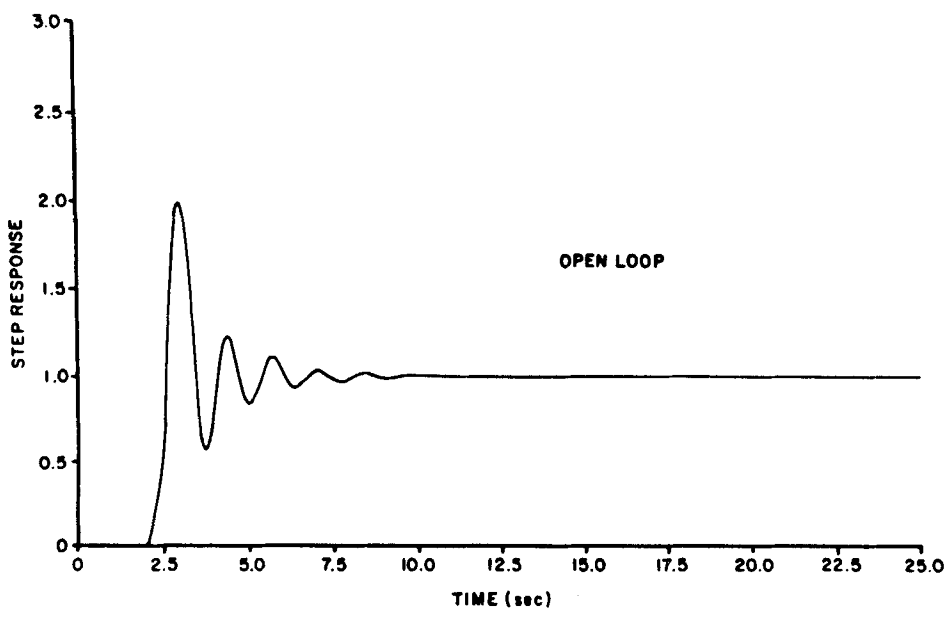


**Figure 7. Open-loop step response, example 2.**

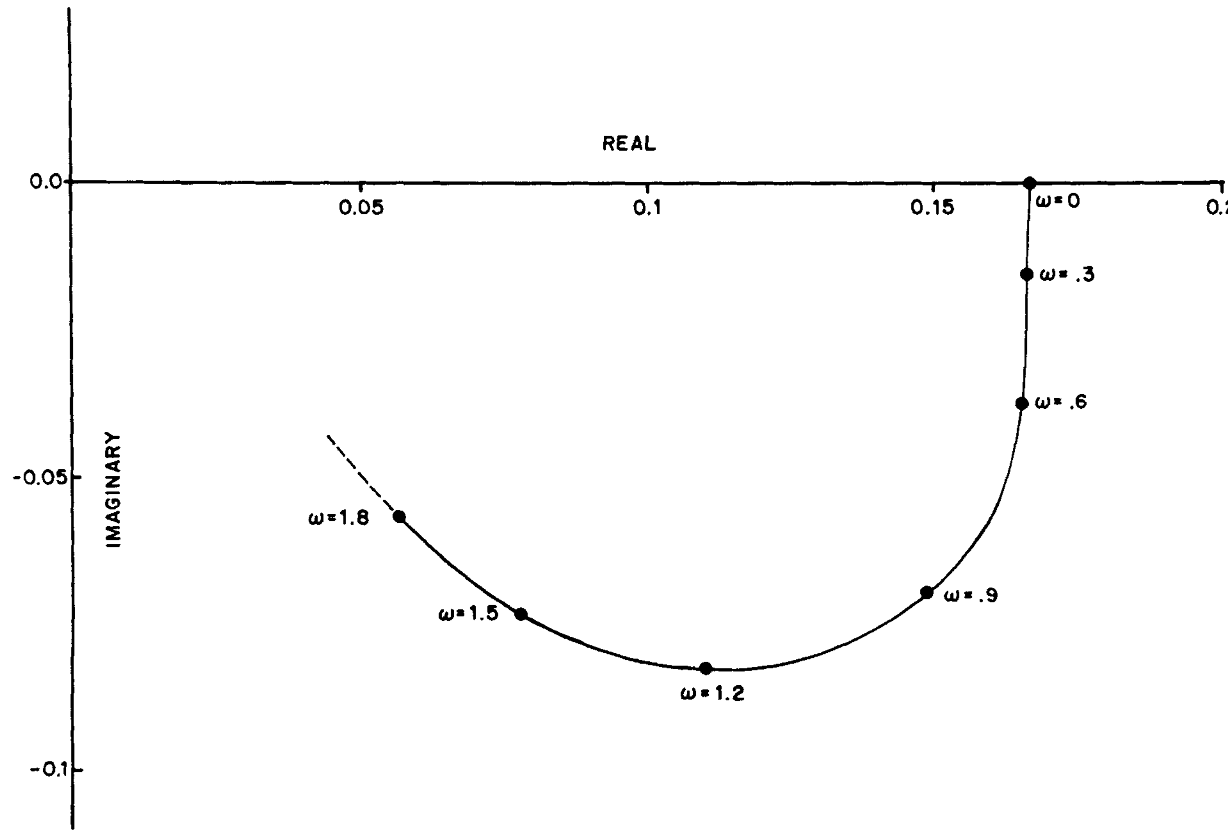


**Figure 8. Nyquist plot of $L(s)$, example 2.**

the dominant ½ structure. The resulting controller is:

$$K\left(\frac{2}{3}\right) = \frac{0.1667}{s} \times \frac{1 + 0.4953s + 0.3016s^2}{1 + 0.8037s + 0.6168s^2}$$

$$= \frac{0.0815}{s} \times \frac{s + 0.821 \pm j1.625}{s + 0.6514 \pm j1.094}$$

Simulation showed that $K(2/3)$ can make the step response match that of the reference model almost exactly. However, its additional complexity (due to the complex poles and zeros) may not be worthwhile since the improvement sought in the step response is not significant.

## Conclusions

This paper furnishes an efficient method to obtain low-order robust controllers for process models that may be of high order and that may involve significant time delay. The computer software is simple and it can be accommodated readily in small computers.

The computational effort involved is essentially insensitive to the order of the original process model since the method needs only its frequency response information. Also, irrational plant transfer functions can be used directly with no sacrifice in accuracy. The method is applicable to both minimum and nonminimum phase process transfer functions. However, in its present form the method cannot be applied to open-loop unstable models. In the case of nonminimum phase models involving dominant right half $s$-plane zeros, the reference model will have to be chosen carefully to include the RHP zeros (Sain et al., 1981). This approach is found to be extremely effective in various designs accomplished by the authors.

The main attribute of the method is that it allows the designer to quickly visualize the structural complexity of the controller to produce a specified step response for the closed-loop system. This enables the designer to make enlightened decisions on time and frequency domain design requirements while making the controller structure simple for practical implementation.

It can be shown readily that the overall controller (including the process model) in the IMC scheme is identical to the ideal controller in Eq. 1 if the reference model is equal to the filter employed in the IMC. However, the controller implementations are drastically different. The advantage of the method in this paper becomes clear in example 2. It illustrates that a designer need not be overwhelmed when faced with a high-order and oscillatory process model. The possibility of a low-order controller becomes apparent as soon as its frequency response is computed.

It is interesting to note that the response characteristics of the process models in examples 1 and 2 are vastly different. The controllers, however, make their respective closed-loop step responses match almost exactly. Thus the problem of making the subsystems of a large system have mutually compatible dynamic characteristics has been effectively solved.

A direction for further work is to extend the design methodology to multiple-input/multiple-output (MIMO) systems. One option is to formulate the equivalent of Eq. 1 in terms of transfer function matrices (Sain and Peczkowski, 1985). Alternatively, a

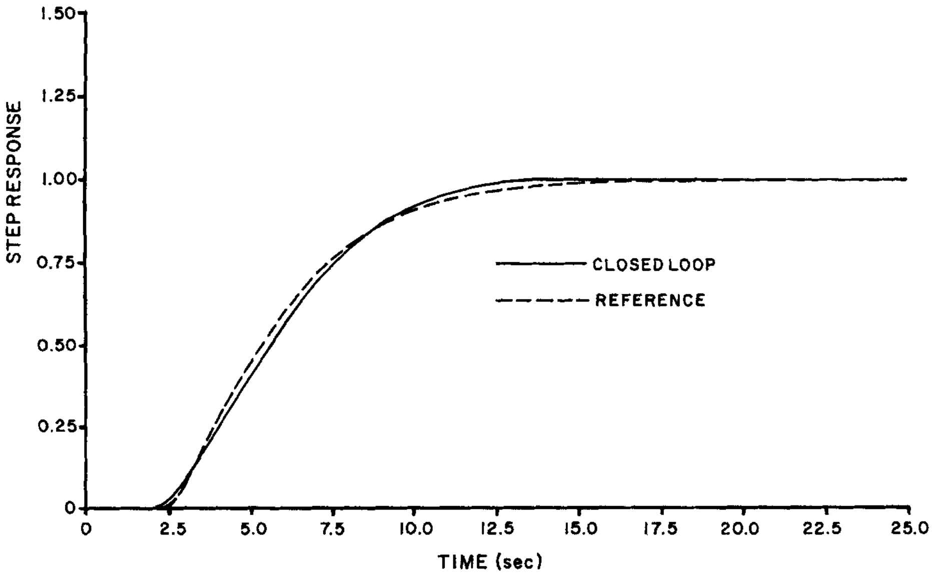


**Figure 9. Closed-loop and reference responses, example 2.**

matrix precompensator can be designed to approximately diagonalize the process model (Rosenbrock, 1974; Kidd, 1986). Independent single-input/single-output (SISO) controllers can then be used for each input-output pair. Once adapted to the appropriate topology, the methodology of this paper can be utilized advantageously. Further research is underway to explore these avenues.

## Notation

$a$ = coefficient of controller transfer function, Eq. 5
$A$ = rational part of reference model
$b$ = coefficient of controller transfer function, Eq. 5
$B$ = defined by Eq. 2
$C$ = output
$G$ = transfer function
$H$ = closed-loop system transfer function
$J$ = index for error minimization
$K$ = controller transfer function
$L$ = part of controller transfer function, Eq. 4
$m$ = degree of numerator of $L$, Eq. 5
$n$ = degree of denominator of $L$, Eq. 5
$N$ = number of frequency points
$p$ = integer, Eq. 8
$P$ = numerator of $L(s)$
$Q$ = denominator of $L(s)$
$R$ = input
$s$ = Laplace transform variable
$t$ = time
$T$ = time constant
$u$ = feedback control law
$W$ = frequency-dependent weight function
$X$ = state vector

### *Greek letters*

$\Delta$ = difference between reference and closed-loop transfer functions
$\epsilon$ = frequency domain error, Eq. 6
$\Gamma$ = state feedback coefficient vector
$\eta$ = index of quality of approximation
$\theta$ = time
$\nu$ = number of poles at origin in $K(s)$
$\tau$ = time delay, time constant
$\phi$ = vector function of time
$\omega$ = frequency, rad/s

### *Subscripts*

$D$ = derivative part
$I$ = integral part
$p$ = process
$R$ = reference

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